Олимпиада ЮМШ 2020-21 года

Код участника: 1 5 1 2

Лист подготовки устного ответа

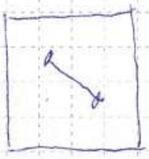
Лист № 1

Фиксация санитарных выходов:

1 выход:		возвращение:	
2 выход:		возвращение:	
3 выход:		возвращение:	
4 выход:		возвращение:	
5 выход:		возвращение:	

Время окончания: 14.25

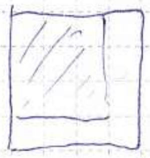
Всего листов: 6

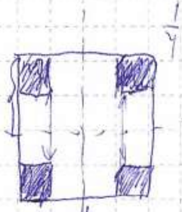
2 

A

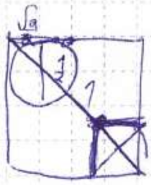
$S(A)$

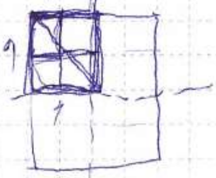
$S(A) \geq 1$



21  $S=4$

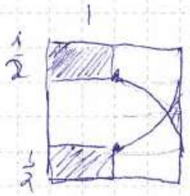
2



2.2 

≥ 2

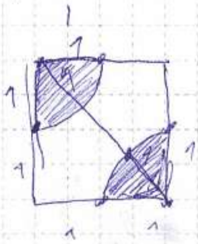
$\frac{1}{2}$



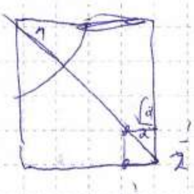
$\frac{1}{2}$

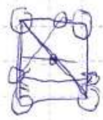
$S = 2\sqrt{2} - \frac{\sqrt{2}}{2} - \frac{1}{2} - 1.5 = \frac{3\sqrt{2}-3}{2} =$

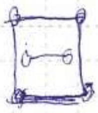
$\frac{\pi R^2}{4} = \frac{\pi}{4}$



$\frac{1}{4}$  $\frac{1}{4}\sqrt{2} = \frac{\sqrt{2}}{4}$







S

$\frac{\pi}{4} + \frac{1}{4}\ V\ 1$

$2\sqrt{2} - 2\ V\ 1$

$(\pi + 1)\ V\ 4$

$2\sqrt{2}\ V\ 3$

$2\sqrt{2} - 1 - \frac{\sqrt{2}}{2}\ V\ 1$

8

$4\sqrt{2} - 2 - \sqrt{2}\ V\ 2$

$3\sqrt{2}\ V\ 4$

$2\sqrt{2}\ V\ 1 + \frac{\sqrt{2}}{2}$

$9 \cdot 2\ V\ 16$

$4\sqrt{2}\ V\ 2 + \sqrt{2}$

$S =$

$3\sqrt{2}$

3.

$n \geq 10$

$n \leq n! - 4^n \leq 4n$

$2^8 \cdot 10079 \leq$

учесть что точно можно найти

$n! - 4^n \geq n$

$n! - n - 4^n \geq 0$

$n! - n \geq 4^n$

$10! - 10 \geq 4^{10}$

$n \geq 10$

$n = 10$

$n! - 4^n \leq 4n$

$11! - 11 \geq 4^{11} \cdot 4$

$2^8(21 \cdot 25 \cdot 27 - 16^3) \cdot 10$

$10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 - 10 =$

$= 10(9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 - 1)$

2^8

$21 \cdot 25 = \frac{2100}{4} =$

$= 525$

$7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \vee 4^7$

$7 \cdot 6 \cdot 5 \cdot 3 \cdot 2 \vee 4^6$

$525 \cdot 27$

$7 \cdot 3 \cdot \cancel{2} \cdot 5 \cdot 3 \cdot \cancel{2} \vee 2^{12}$

$7 \cdot 3^2 \cdot 5 \vee 2^{10}$

525×27: 3675, 1050, 14175

16×16: 96, 16, 256

$10!$: $10 \cdot 9$

$10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \vee 2^{20}$

$5 \cdot \cancel{2} \cdot 3^2 \cdot \cancel{2^3} \cdot 7 \cdot \cancel{2} \cdot 3 \cdot 5 \cdot \cancel{2^2} \cdot 3 \cdot \cancel{2} \vee 2^{20-8}$

$5 \cdot 3^2 \cdot 7 \cdot 3 \cdot 5 \cdot 3 \vee 2^{12}$

$5^2 \cdot 7 \cdot 3^4 \vee 2^{12}$

$2^4 = 16$

$21 \cdot 5^2 \cdot 3^3 \quad \vee 16^3$

$16^3 =$

$16 \cdot 16$

$2^8 \cdot 21 \cdot 25 \cdot 27 \vee 16^3 \cdot 2^8$

$\vee 16^3 \cdot 2$

256×16: 1536, 256, 4096

$14175 - 4096 =$

$= 10079$

Код участника: 1512

Лист подготовки ответа

Лист № 2

$n! - 4^n > 4n$

$(n+1)! - 4^n \cdot 4 \vee 4(n+1)$

$n! \cdot n - 4^n \cdot 4 \vee 4n+4$

$n! \cdot 4 + n!(n-4) - 4^n 4 - 4n \vee 4$

$\underbrace{4(n! - 4^n - n)}_{>0} + \underbrace{n!(n-4)}_{>0} - 4 \vee 0$

3.2

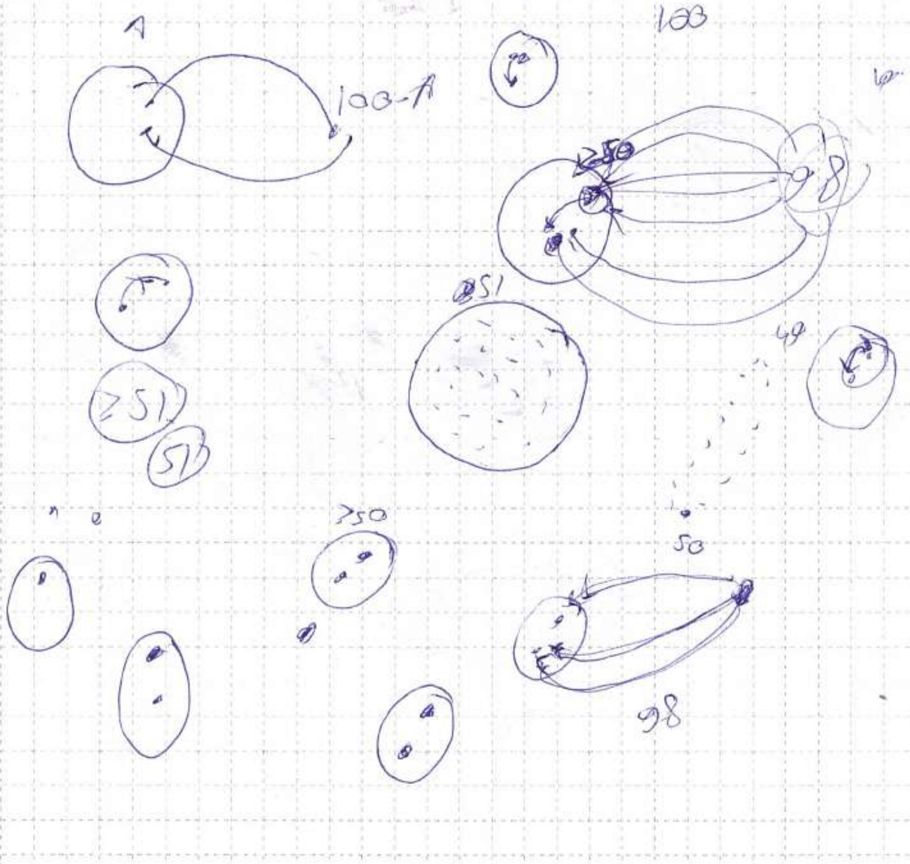
$n \le n! - k^n \le kn$

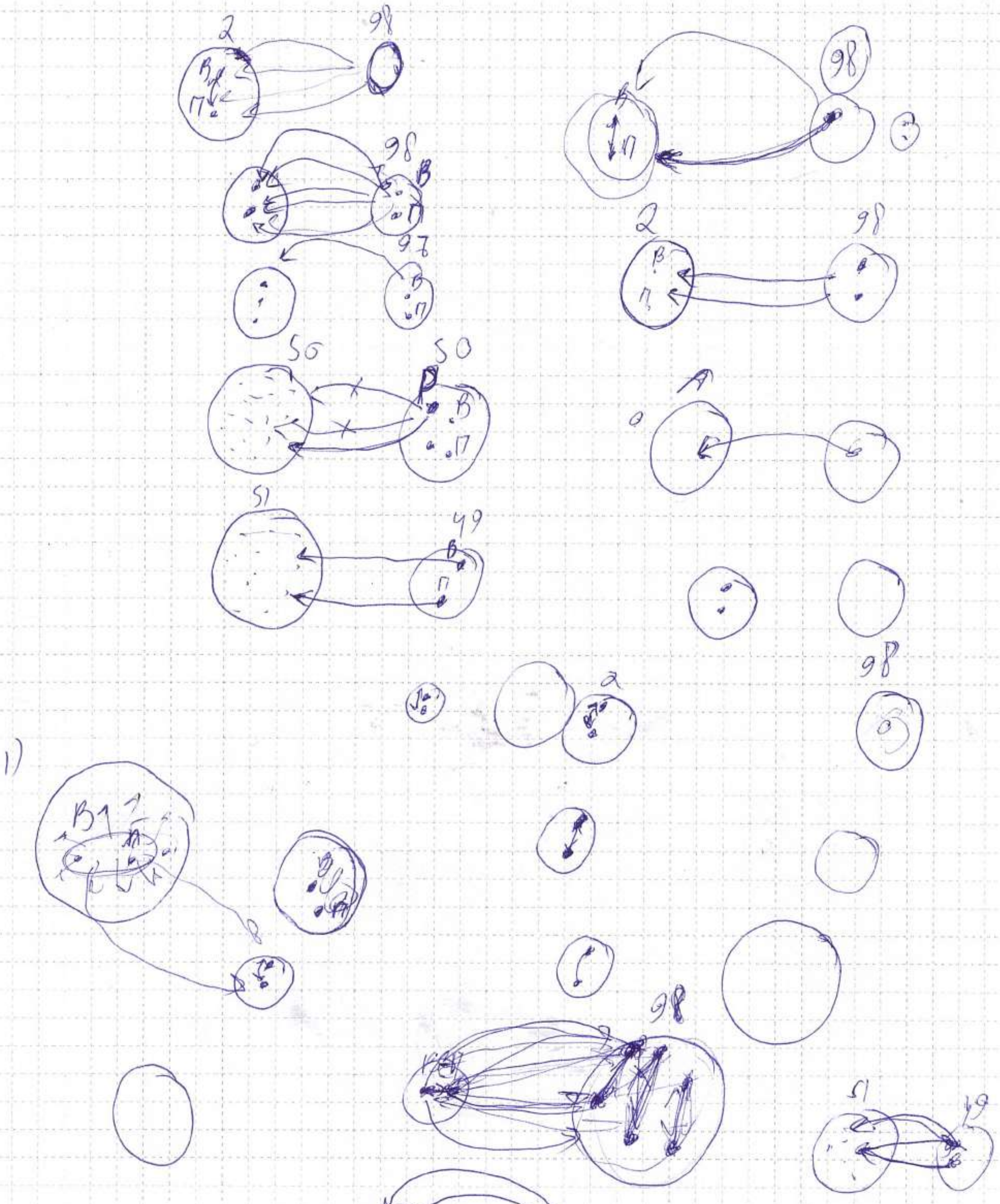
$kn \ge n! - k^n$

$10k \ge 10! - k^{10}$

$k \le 4$

1.

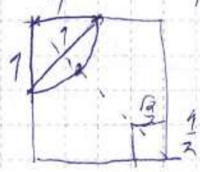




2
98
В
П
98
В
П
97
В
П
50
50
В
П
51
49
В
П
2
98
В
П
98
2
98
1)
98
51
49

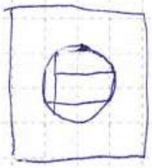
Код участника: 1512 **Лист подготовки ответа** Лист № 3

2.1



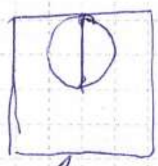
$\pi \vee \frac{2}{2-\sqrt2}$ = $\cancel{2\sqrt2}$ $\frac{2(2+\sqrt2)}{4-2}$ = $3\vee\frac{2}{2-\sqrt2}$ $6-3\sqrt2 \vee 2$

$S=2\sqrt2-1-\frac{\sqrt2}{2}$ = $=2+\sqrt2$



$d<1$ $2\pi-\pi\sqrt2 \vee 2$

$9+2\vee12$



$2\pi-2\vee\pi\sqrt2$

$4\pi^2-8\pi+4\vee2\pi^2$ $\frac{1}{4}$

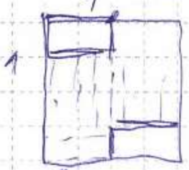
$\frac{1}{2}$ $2\pi^2+4\vee8\pi$

$\pi^2+2\vee4\pi$

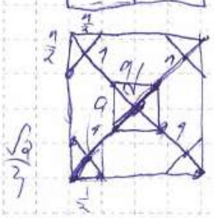
$\frac{\pi}{4}(4-2\sqrt2)\vee1$

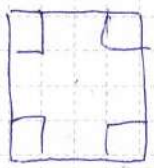
$\pi(4-2\sqrt2)\vee4$

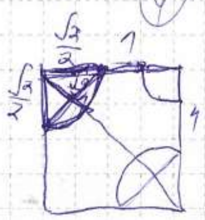
$\pi(2-\sqrt2)\vee2$



$\frac{1}{4}$ 1 $\frac{1}{4}$







$S=\frac{\sqrt2}{2}\cdot\frac{\sqrt2}{2}\cdot\frac12=\frac14$

$\frac14=\frac12\cdot h\cdot 1$

$h=\frac12$

$a\sqrt2=8 \quad \vee = 2\sqrt2-2-\frac{\sqrt3}{2}$

$\pi^2-4\pi+2\vee0$

$\cancel{\frac{2\pm}{4}}$

$D=4^2-4\cdot2=$

$=16-8=8$

$\pi_1=\frac{4\pm2\sqrt2}{}$

$a=2-\sqrt2-\frac12=\frac32-\sqrt2=\frac{3-2\sqrt2}{2}$

$a^2=\frac{(3-2\sqrt2)^2}{4}=$

$S=\frac{\pi R^2}{4}=$

$=\frac{\pi}{4}\cdot\frac12=\frac{\pi}{8}$

$\pi\vee2+\sqrt2$

$S=2\sqrt2-\sqrt2=1\sqrt2$

$S=\frac{\pi}{4}\cdot\frac{(2-\sqrt2)^2}{4}=$

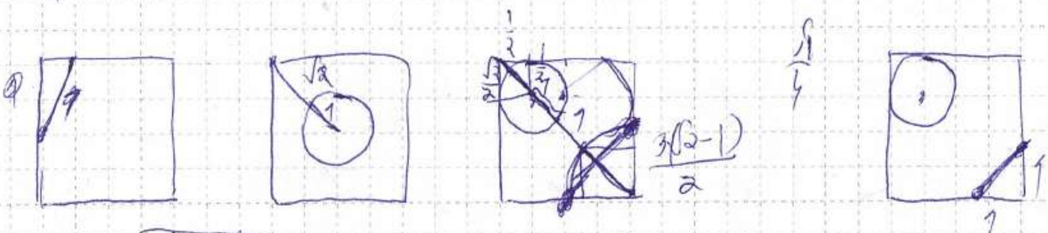
$=\frac{\pi}{16}(4-4\sqrt2+2)=$

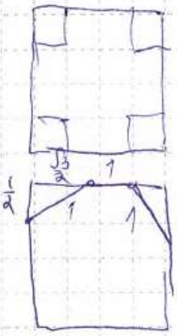
$=\frac{\pi}{16}(6-4\sqrt2)$

$2-2(\frac{\sqrt2}{2})=2-\sqrt2$

$\cancel{S_{18}}$ $R=2-1-\frac{\sqrt2}{2}=1-\frac{\sqrt2}{2}=\frac{2-\sqrt2}{2}$

$S_0=\frac{\pi}{4}+\frac{\pi}{8}(6-4\sqrt2)=\frac{\pi}{4}(1+\frac12(6-4\sqrt2))=\frac{\pi}{4}(1+3-2\sqrt2)=\frac{\pi}{4}(4-2\sqrt2)$





$2\sqrt{2} - \frac{\sqrt{2}}{2} - \frac{1}{2} - 1 =$

$= \frac{3\sqrt{2}}{2} - \frac{3}{2} = \frac{3\sqrt{2}-3}{2} = \frac{3(\sqrt{2}-1)}{2}$;

$S = \frac{1}{4} \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{8}$

$a\sqrt{2} = \frac{3(\sqrt{2}-1)}{2}$

$\sqrt{1-\frac{1}{4}} = \frac{\sqrt{3}}{2}$ $d =$

$a = \frac{3}{\sqrt{2}}(\sqrt{2}-1)$

$2 \vee \frac{3}{2}\sqrt{2}$

$4 \vee 3\sqrt{2}$

$16 \vee 9.$

$\frac{3(2-\sqrt{2})}{2} = 3 - \frac{3}{2}\sqrt{2} \vee 1$

$S = a^2 = \frac{9}{2}(\sqrt{2}-1)^2$

$580+6\cdot58=$

$=580+6\cdot50+6\cdot8=$

$=580+300+48=$

$=880+48=928$

$S = \frac{\pi R^2}{4} = \frac{\pi}{4} \cdot \frac{9}{4}(\sqrt{2}-1)^2 = \frac{9}{16}\pi(\sqrt{2}-1)^2$

$\frac{9}{16}\pi(\sqrt{2}-1)^2 + \frac{\pi}{4} \vee 1$

3,14159

$9\pi(2-2\sqrt{2}+1)+4\pi \vee 16$

$\pi(27-18\sqrt{2}+4) \vee 16$

$\pi(31-18\sqrt{2}) \vee 16$

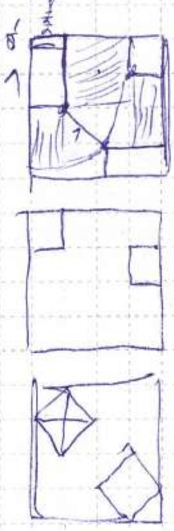
$\pi \vee \frac{16}{31-18\sqrt{2}} = \frac{16(31+18\sqrt{2})}{31^2-18^2\cdot 2} = \frac{16}{313}(31+18\sqrt{2}) <$

$< \frac{16}{313}\left(31+18\cdot\frac{3}{2}\right) =$

$= \frac{16}{313}(31+27) =$

$= \frac{16}{313}\cdot 58 =$

$= \frac{16\cdot 58}{313} = \frac{928}{313}$

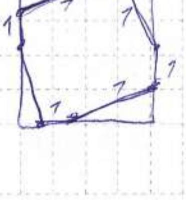


$(30+1)^2 = 900+60+1 = 961$

$(20-2)^2 = 400-80+4 =$
$= 324$
648

$961-648=$
$=313$

$2\cdot\sqrt{2}$

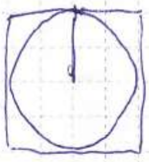


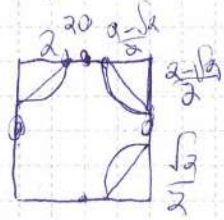
Код участника: 1 5 1 2 Лист подготовки ответа Лист № 4

3.1

$n! - 4^n \vee 4n \quad n=10$

$10! - 4^{10} \vee 40$





$10! = 10\cdot9\cdot8\cdot7\cdot6\cdot5\cdot4\cdot3\cdot2 =$

$= 5\cdot2\cdot3^2\cdot2^3\cdot7\cdot3\cdot2\cdot5\cdot2^2\cdot3\cdot2 =$

$= 5^2\cdot2^8\cdot3^4\cdot7$

$2^8(5^2\cdot3^4\cdot7 - 2^{12}) \vee 40$

$2^8(25\cdot21\cdot27 - 16^3) \vee 40$

$67\cdot64 = (60+7)(60+4) =$

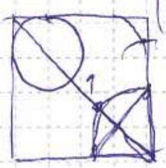
$= 3600+240+420+28 =$

$= 4260+28 = 4288$

$\frac{\pi}{4} + \frac{\pi}{2}\cdot\frac{(2-\sqrt2)^2}{4} \vee 1$

$2\pi + \pi(2-\sqrt2)^2 \vee 8$

$2\pi + \pi(6-4\sqrt2) \vee 8$



$\frac{6-3\sqrt2}{2} \not< 1$

$6-3\sqrt2 \not< 2$

$4 \not< 3\sqrt2$

$\frac{2\sqrt2-2}{2}$ длина $\pi \vee$ ширина

$\times\, 4288 \quad 27 \quad 6 \quad 56$

$\pi(2-\sqrt2) \vee 2$

$\pi \vee \frac{2}{2-\sqrt2} =$

$= \frac{2(2+\sqrt2)}{2}$

$= 2+\sqrt2$

$n! - 4^n > 4n$

$(n+1)! - 4^{n+1} \vee 4(n+1)$

$n!(n+1) - 4\cdot4^n \vee 4n+4$

$n!(n-3) + n!\,4 - 4\cdot4^n - 4n - 4 \vee 0$

$4(n! - 4^n - n) + n!(n-3) - 4 \vee 0$

$1 - \frac{\sqrt2}{2} = \frac{2-\sqrt2}{2}$

$8\pi - 4\sqrt2\pi \vee 8$

$2\pi - \pi\sqrt2 \vee 2$

$1 \le (n-1)! - \frac{k^n}{n} \le k$

$0 < (n-1)! - \frac{4^n}{n} \le k$

$(n-1)! > \frac{k^n}{n}$

$(n-1)!\cdot n > k^n$

3.2

$n > k \quad n = k+p$

$(k+p)! - k^{k+p} \vee k(k+p)$

$(k+p)!($

$4288\cdot7 = 28000$

$+\,1400+560+56 =$

$= 29400+1960+56 =$

$= 29400+2016 =$

$= 31416$

$S = \frac{\pi}{4}\cdot\frac{1}{2} = \frac{\pi}{8}$

$10! - 6^{10} = 2^{10}\cdot3^{10}$

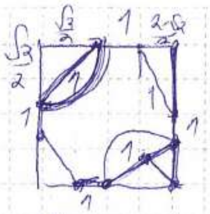
$2^8\cdot3^4(5^2\cdot7 - 2^2\cdot3^6)$

$\frac{1{,}1416\cdot16}{31-18\sqrt2} = \frac{114\cdot16}{100(31-18\sqrt2)} = \frac{114\cdot16(31+18\sqrt2)}{100\cdot313} - \frac{\pi}{2}\cdot\frac{(2-\sqrt2)^2}{4}$

$= \frac{57\cdot16(31+18\sqrt2)}{50\cdot313} = \frac{57\cdot16(\cdot54}{50\cdot313} =$

$900\cdot30 = 2700$

$= \frac{67\cdot32\cdot54}{31300} = \frac{27\cdot67\cdot64}{31300}$



$1-\frac{\sqrt{3}}{2}=\frac{2-\sqrt{3}}{2}$ | $S=\frac{1}{2}+6-4\sqrt{2}+2\cdot\frac{2-\sqrt{2}}{2}(\sqrt{2}-1)=\frac{1}{2}+6-4\sqrt{2}+2\sqrt{2}-\cancel{2}-2+\sqrt{2}=$

2.1

$\sqrt{1-\frac{4-4\sqrt{2}+2}{4}}=\sqrt{1-\frac{6-4\sqrt{3}}{4}}=\sqrt{\frac{4+4\sqrt{2}-6}{4}}=\sqrt{\frac{4\sqrt{2}-2}{4}}=$

$S=\frac{1}{2}\cdot\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{2}}{2}=\frac{1}{4}$

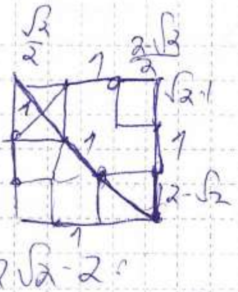
$=\frac{\sqrt{4\sqrt{2}-2}}{2}\ V$

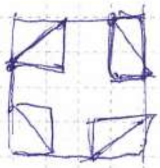
$=\frac{1}{2}+2-\sqrt{2}\ V\ 1$

$1-\frac{\sqrt{4\sqrt{2}-2}}{2}$

$2\sqrt{2}+2$

$(2\sqrt{2}+1)-a$





6

$S=\frac{1}{4}\cdot\frac{1}{2}=\frac{1}{8}$

$2\sqrt{2}-2=$

$2-1-2+\sqrt{2}=$
$=\sqrt{2}-1$

$S=\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{2}}{2}=\frac{1}{2}$

$S=\left(\frac{2-\sqrt{2}}{2}\right)^2=\frac{4-4\sqrt{2}+2}{4}=\frac{6-4\sqrt{3}}{4}=\frac{3-2\sqrt{3}}{2}$

$\frac{1+3-2\sqrt{2}}{2}=\frac{4-2\sqrt{3}}{2}=2-\sqrt{2}$

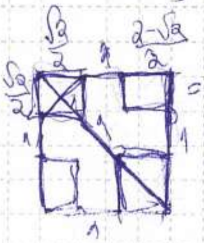
$a\sqrt{2}=2\sqrt{2}-2$

$a=2-\sqrt{2}$

$S=\frac{3}{4}$

$S_2=\frac{2-\sqrt{2}}{2}\cdot\frac{\sqrt{4\sqrt{2}-2}}{2}=$

$a^2=6-4\sqrt{2}$



$2\sqrt{2}-2\ V\ 1$

$=\frac{(2-\sqrt{2})\sqrt{4(\sqrt{2}-2)}}{4}$

$(2-\sqrt{2})^2\ V\ \frac{1}{2}$

$2-\sqrt{2}\ V\ \frac{1}{\sqrt{2}}$

$a=2-\sqrt{2}$

$\frac{1}{2}+6-4\sqrt{2}\ V\ 1$

$a^2=6-4\sqrt{2}$

$6-4\sqrt{2}\ V$

$2\sqrt{2}-1\ V\ 1$

$S_3=\frac{\sqrt{2}-1}{\sqrt{2}}\cdot(\sqrt{2}-1)=\frac{(\sqrt{2}-1)^2}{\sqrt{2}}$

$\frac{3}{2}\ V\ \sqrt{2}$

$S=\frac{1}{2}+(2-\sqrt{2})^2+\frac{(\sqrt{2}-1)^2}{\sqrt{2}}\cdot 2=\frac{1}{2}+(2-\sqrt{2})^2+\sqrt{2}(\sqrt{2}-1)^2=$

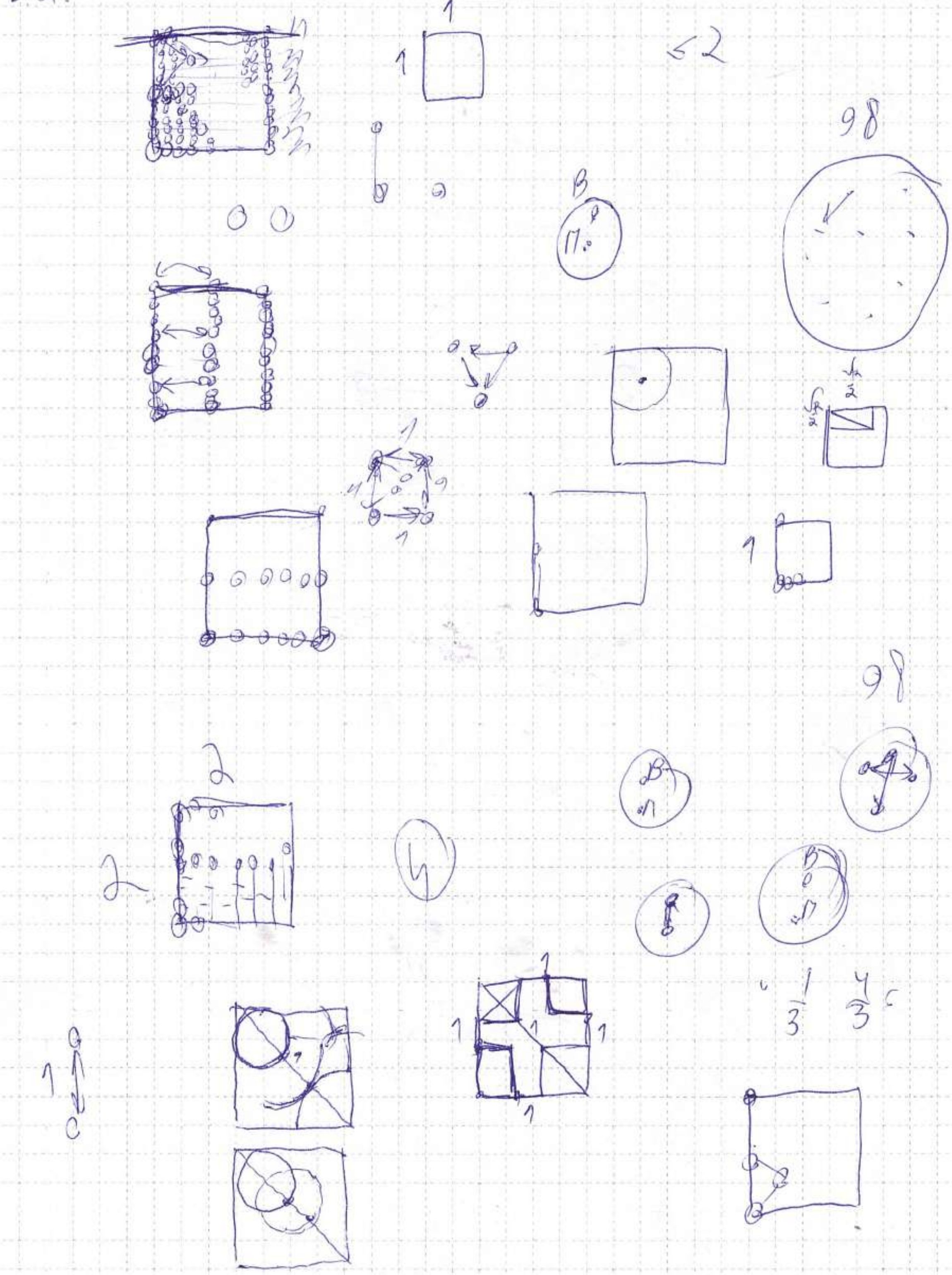
$=\frac{1}{2}+6-4\sqrt{2}+\sqrt{2}(2-2\sqrt{2}+1)=\frac{1}{2}+6-4\sqrt{2}+3\sqrt{2}-4=$
$=\frac{1}{2}+2-\sqrt{2}\ V\ 1$

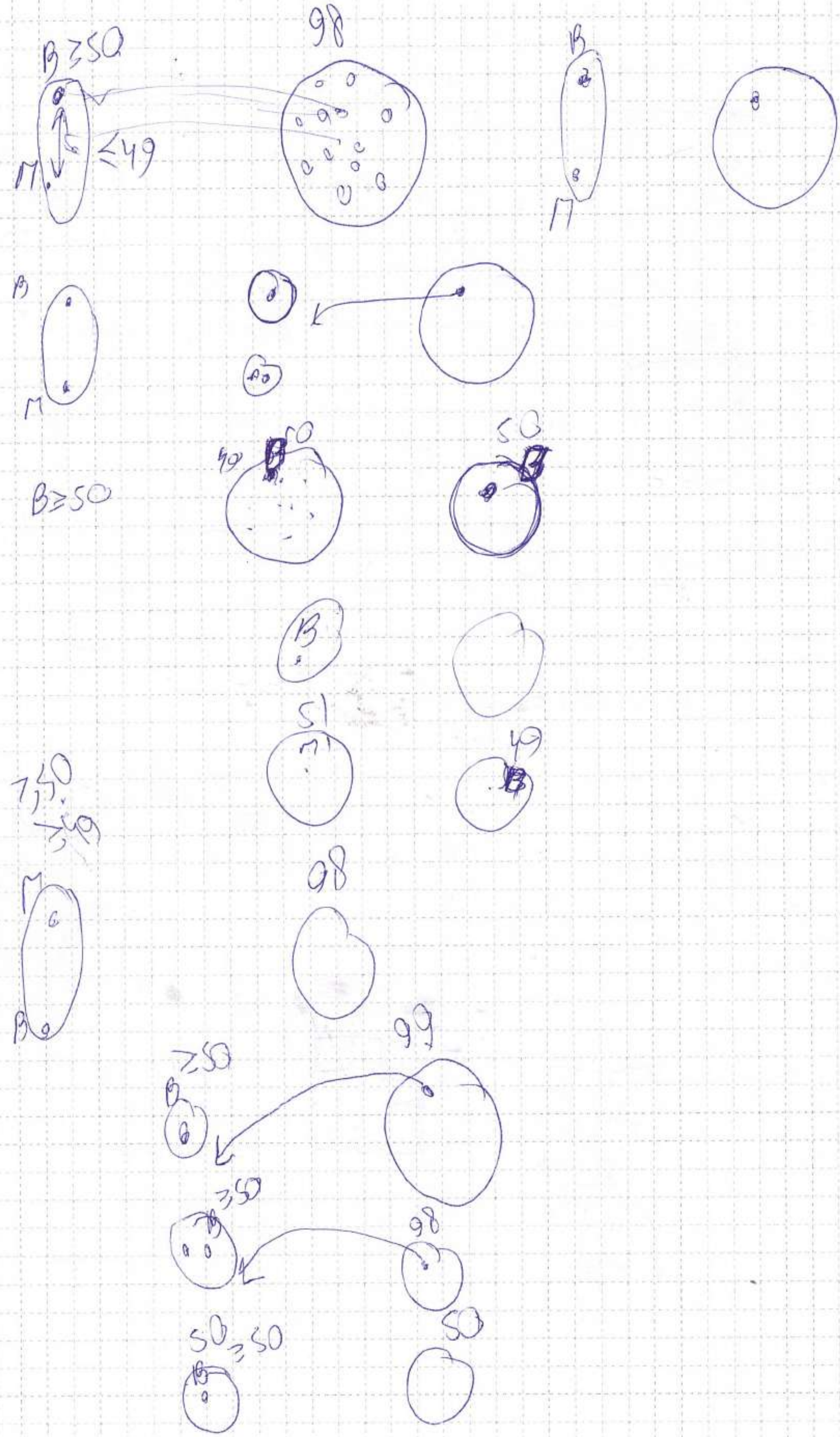
Код участника: 1512

Лист подготовки ответа

Лист № 5

32.





B ≥ 50
98
≤ 49
M
B
M
B
M
B ≥ 50
50
50
B
51
49
98
M
B
99
≥ 50
B
≥ 50
98
50 ≥ 50
50

Код участника: 1512 **Лист подготовки ответа** Лист № 6

$\pi \vee \frac{1{,}14\cdot 16}{31-18\sqrt2}$

1,41

$\frac12 + (2-\sqrt2) \vee 1{,}14$

0,58

$0{,}5 + 0{,}58 = 1{,}08$

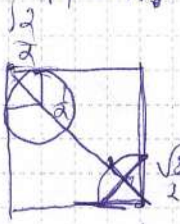
$\frac{\pi}{4}\left(\frac{\pi}{4} + \frac{\pi}{16}(27-18\sqrt2)\right) \vee 1{,}14$

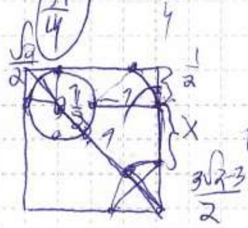
$\frac{\pi}{16}(4+27-18\sqrt2) \vee 1{,}14$

$n, k \in N$

$\frac{\pi}{4}$

$n \le n! - k^n \le kn$





$\frac{\pi}{4} + x \vee 1{,}14$

$S = \frac{\pi}{4}\cdot\frac12 = \frac{\pi}{8}$

$n! - k^n$

$(n+1)k$

$2\sqrt2$

3,14

$\frac{6-3\sqrt2}{2} \vee 1$

$\frac{\pi}{4}$

$2\sqrt2 - \frac{\sqrt2}{2} - \frac12 - 1 =$

$= \frac{3\sqrt2-3}{2}$

$\frac{\pi}{16}(31-18\sqrt2) \vee 1{,}14$

$6-3\sqrt2 \vee 2$

$4 \vee 3\sqrt2$

$x \vee 1{,}14 - \frac{\pi}{16}(31-18\sqrt2)$

16

$S = \frac{\pi}{4}\cdot\frac{18-18\sqrt2+9}{4} =$

$= \frac{\pi}{16}(27-18\sqrt2)$

$2\sqrt2 - \frac{\sqrt2}{2}\cdot 2 - \frac12 =$

$= 2\sqrt2 - \sqrt2 - \frac12 = \sqrt2 - \frac12$

$\frac{\pi}{16}(27-18)$

$x + \frac12 + \frac{3\sqrt2-3}{2} = 2$

$2x + 1 + 3\sqrt2 - 3 = 4$

$2x = 6 - 3\sqrt2$

$x = \frac{6-3\sqrt2}{2} \vee 1$

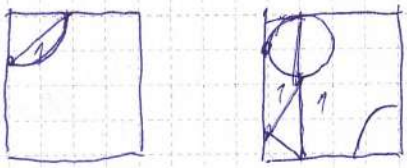
$= \sqrt2 - \frac{\sqrt2}{2} - \frac12 \cdot 1 =$

$= \frac{3\sqrt2-3}{2} = \frac32(\sqrt2-1)$

$6-3\sqrt2 \vee 2$

$4 \vee 3\sqrt2$

$\underline{27 \cdot 4288}$
31306



$t \geq 1$
$k = n - t$

$50 \cdot 50$
$2500 \cdot 50 =$
$=$

$n \geq k$

$n = k + t \geq 10$

$k + t \leq (k+t)! - k^{k+t} \leq k(k+t)$

$0 \leq (n-1)! - \frac{k^n}{n} \leq k$

$(n-1)! \geq \frac{k^n}{n}$

$k = 2, 3, 4, 5, 6, 7$

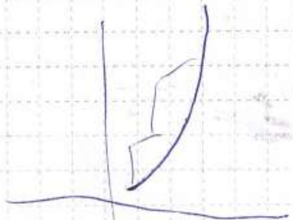
$n! > k^n$

$(n-1)! - \frac{k^n}{n} \leq k < \frac{k^n}{n}$

$(n-1)! \leq k\left(1 + \frac{k^{n-1}}{n}\right)$

$(n-1)! \leq \frac{2k^n}{n}$

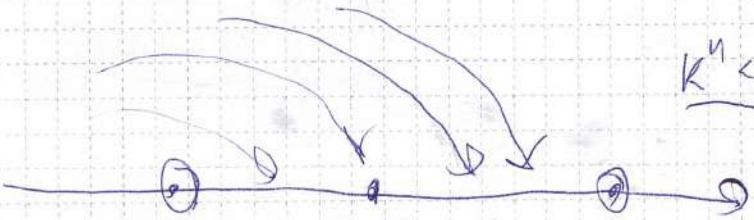
$n \geq k$



$n \leq k$

$k^n < n! \leq 2k^n$

$\underline{k^n < n! < 2 \cdot k^n}$



k^n $\qquad$ $2 \cdot k^n$

решений не более 1

$n! < k^{n-1} \cdot n = k^n \cdot \frac{n}{k} < k^n$

$n! > k^n$
$\Downarrow$
$n! > 2 \cdot k^n$

$\begin{cases} (n-1)! < k^{n-1} \\ n! > k^n \end{cases}$
$\Downarrow$
$n! > k^n + kn$

$n = k + t$

$k^n\left(\frac{k+t}{k}\right) = k^n + k^{n-1}t$

k

Код участника: 1512 Лист подготовки ответа Лист № 7

$0 < n! - k^n$

$n! - k^n < k^n$

$k^n < n! < 2 \cdot k^n$

$kn - n = n(k-1)$

$n! - k^n < 0$

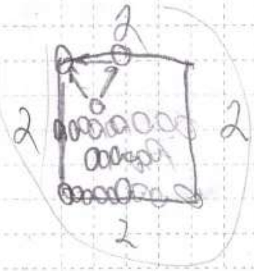
$(n+1)! - k^{n+1} > ?$

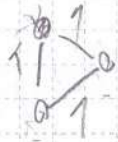
$n! \cdot (n+1) - k^n \cdot k > ?$

$n! \cdot k + n!(n+1-k) > ?$

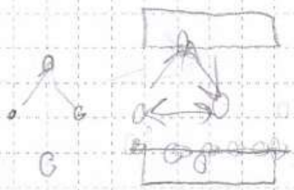
$k(n! - k^n) + n! > kn$

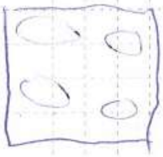
$n! - kn > k(k^n - n!)$

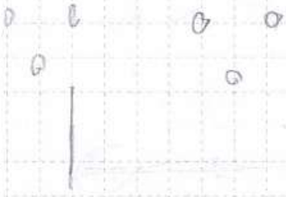




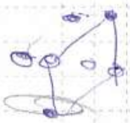
$$\frac{4}{3}$$

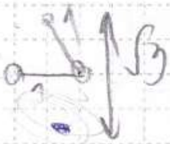


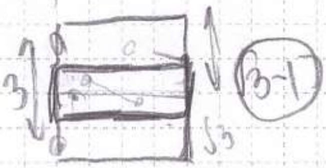


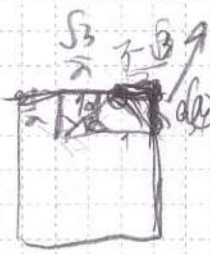


0,035





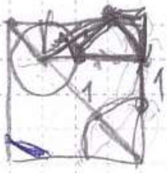


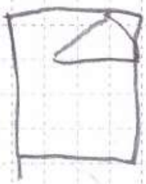


$$\frac{5-3\sqrt{2}}{2}$$

0,38

$$S = \frac{\pi}{4} = 0{,}25\pi$$





$$R = 2\sqrt{2} - \frac{\sqrt{2}}{2} - \frac{1}{2} - 1 = \frac{3\sqrt{2}-3}{2} \approx 0{,}62$$

$$S = \frac{\pi}{4}\, 0{,}36 = \pi \cdot 0{,}09 = 0{,}09\pi$$

$0{,}34\pi$

$1{,}068$

$$1 - \frac{3\sqrt{2}-3}{2} = \frac{5-3\sqrt{2}}{2}$$

$$R = 2 - \frac{\sqrt{3}}{2} - \frac{1}{2} = \frac{4-\sqrt{3}-1}{2} = \frac{3-\sqrt{3}}{2}$$

$$S = \frac{(3-\sqrt{3})(5-3\sqrt{2})}{4} =$$

$$= \frac{15-9\sqrt{2}-5\sqrt{3}-3\sqrt{6}}{8} =$$

$$S_0 = \frac{\alpha}{360} \cdot 1 =$$